Dispersive Self-$Q$-Switching in DFB Lasers—Theory Versus Experiment

Uwe Bandelow, Hans-Jürgen Wünsche, Member, IEEE, Bernd Sartorius, and Martin Möhrle

Abstract—The single-mode model of dispersive self-$Q$-switching is extended to lasers containing a phase tuning section. The parameter set used for modeling is taken from independent measurements on existing self-pulsating devices. Detuning of the Bragg wavelengths by current induced heating is found, and this effect is included in the model. Calculated self-pulsation characteristics were compared quantitatively with experimental results on the device. A very good correspondence between theory and experiment is obtained, e.g., for conditions generating self-pulsations and for the frequency-current dependence. Dispersive self-$Q$-switching thus is confirmed as responsible mechanism for the high frequency DFB type self-pulsations. The modeling further shows that the delay between the stimulated emission within the device and the radiation of photons from the facets plays an important role for keeping the pulsations running.

Index Terms—Distributed-feedback lasers, dynamics, indium materials/devices, modeling, optical pulse generation, pulsed lasers, $Q$-switched lasers, semiconductor lasers

I. INTRODUCTION

FIRSTLY, in 1992, high-speed self-pulsations (SP) in two-section DFB lasers have been observed [1]. Up to now there were reported repetition frequencies up to 64 GHz, continuously tunable by the currents [2]. In addition the authors could demonstrate 18 GHz all optical clock recovery using these self-pulsating devices [3]. A further improvement of the device performance has been achieved by integrating a phase tuning section [4], which made the SP’s reproducible and controllable. All-optical clock recovery (BER < $10^{-12}$) could be demonstrated with such devices [5], [6] in wavelength and polarization independent operation.

Modeling results have been published for self-pulsating single- and double-section DFB-lasers, proposing different possible mechanisms responsible for the SP’s. Among these was spatial hole burning (SHB) [7]–[9], dispersive self-$Q$-switching (DQS) [9], [10], [12], and [13], and mode beating [12], [13]. No direct quantitative comparison between modeling and measurements was reported, so far.

In three-section lasers with integrated phase tuning section, the basic features of DQS could be experimentally demonstrated very recently [14]. These are the single-mode nature of DQS on one hand, the typical bias conditions (very asymmetric pumping of the sections) on the other hand and the repetition frequencies of usually around 10 GHz.

In this paper, we present the extension of the single-mode model [10] for DQS in two-section DFB lasers to three-section devices as sketched in Fig. 1. A quantitative comparison of the model with measurements on a corresponding 1.55-$	ext{µm}$ InP RW laser—the same device as in [4], [14]—will be performed. Throughout the paper, the following operation conditions are considered. The outer DFB section is driven close to its gain transparency and acts mainly as a dispersive reflector. The middle DFB section is highly injected and provides the gain for the device. The third section in which active layer and grating are removed, serves as a passive phase tuning section. Accordingly, quantities referring to the reflector, gain and phase sections will be labeled by $r$, $g$, and $p$, respectively.

A brief description of the basic equations of the single-mode model of DQS, which have been derived already in earlier papers [13], [15], is given in Section II. The determination of essential model parameters from measurements follows in Section III. In Section IV, especially the role of the integrated phase tuning section for controlling the DQS will be considered. Further quantitative comparison between theory and experiment will be performed in Section V. A detailed discussion of the DQS-Mechanism follows in Section VI.

II. SINGLE-MODE MODEL

The SP’s generated by the considered devices have been shown to be monomode [4], [14]. Therefore, a single-mode description should be possible, but so far a specific model for single-moded SP’s has been reported only by our group [10]–[15].

The Model is based on the single-mode approximation of the traveling wave equations described in [13], [15], which was firstly applied to self-pulsating two-section DFB-lasers in [10] and confirmed by quantitative comparison with the comprehensive time-domain model in [12]. As there, we ne-
glect the spontaneous emission and assume spatially constant carrier densities in each DFB section, thereby neglecting spatial hole burning effects within a section. The carriers within the passive phase section do not participate in the dynamics but are completely determined by the dc phase current $J_p$. In consequence, the model contains three basic dynamic quantities, the number $S$ of photons within the lasing mode, as well as the numbers $N_x$ and $N_y$ of carriers contained in the two active DFB sections. Their evolution is governed by the rate equations ($k = r, g$)

$$\frac{d}{dt} N_k = \frac{I_k}{e} - \frac{N_k}{\tau} - v_g g_k \Gamma_k S$$  \hspace{1cm} (1)

$$\frac{d}{dt} \left( \sqrt{S K_z} \right) = (G - \gamma) \left( \frac{S}{\sqrt{S K_z}} \right)$$  \hspace{1cm} (2)

with $e$ being the elementary charge, $I_k$ the injection current into section $k$ and $\tau$ the spontaneous lifetime. The sectional gain $g_k$ will be specified in the next section. $\Gamma_k$ is a longitudinal fill factor, i.e., the relative portion of power contained in section $k$:

$$\Gamma_k = \frac{\int_0^L |\Phi|^2 \, dz}{\int_0^L |\Phi|^2 \, dz}, \quad \text{with} \quad |\Phi|^2 = |\Phi_+|^2 + |\Phi_-|^2.$$  \hspace{1cm} (3)

Here, the two component vector $\Phi = (\Phi_+, \Phi_-)$ denotes the appropriately normalized amplitudes of the forward and backward traveling waves of the lasing mode (c.f. [13], [15]). They are solutions of the instantaneous coupled mode equations

$$\left( i \frac{\partial}{\partial z} + \frac{\pi}{\Lambda} - \beta - \kappa \right) \Phi = \frac{\Omega}{v_g} \Phi$$  \hspace{1cm} (4)

subject to the reflecting boundary conditions

$$\Phi_+ = r_0 \Phi_- \text{ at } z = 0$$

and

$$\Phi_- = r_L \Phi_+ \text{ at } z = L$$  \hspace{1cm} (5)

with the amplitude reflectivities $r_0$ and $r_L$ at the end facets of the laser and the group velocity $v_g$. $\Lambda$ denotes the local corrugation period. $\beta$ is the propagation constant of the local waveguide at a fixed central frequency $\omega_0$. It varies with the longitudinal position $z$ according to the material parameters, the carrier densities and the temperatures of the different sections. In the DFB sections, $\beta$ furthermore depends on the time $t$ via the carrier densities. Thus, (4) depends parametrically on $t$. i.e., it describes the lasing mode belonging to the instantaneous carrier distribution. Because the latter one undergoes changes, both the spatial shape of the amplitudes $\Phi$ and the eigenvalue $\Omega$ also exhibit a temporal evolution. The real part of $\Omega$ is the modal optical frequency (relative to $\omega_0$), the imaginary part is the decay rate of the mode. From (4) one obtains easily that this decay rate is equal to the balance $2 \Im m(\Omega) = \gamma - G$ that appears in the photon rate (2). Here,

$$G = v_g \int_0^L 2 \Im m \beta |\Phi|^2 \, dz - \int_0^L |\Phi|^2 \, dz$$  \hspace{1cm} (6)

is the mean net gain (including internal optical losses) of the device and

$$\gamma = \frac{v_g}{2} \left( 1 - |r_0|^2 \right) |\Phi_+(L)|^2 + \left( 1 - |r_L|^2 \right) |\Phi_-(0)|^2$$  \hspace{1cm} (7)

is the loss rate due to the radiation emitted at the facets.

In solving (4), one has to choose that solution with the smallest decay rate, because it represents the lasing mode and is the dominating degree of freedom of the optical field. The neglect of the other modes holds as long as their gain margin to the lasing mode remains large compared with the repetition frequency of the pulsations [13]. This holds for all examples to be presented below with at least 150 ns$^{-1}$ gain margins and around 10-GHz pulsation frequencies. In the stationary case, the single-mode approximation becomes an exact solution of the traveling wave equations without spontaneous emission.

Our photon rate (2) differs from the conventional one mainly by the appearance of $\Phi$, which is the longitudinal analogy to Petermann’s $K$-factor of excess spontaneous emission [16]:

$$K_z = \left| \frac{\int_0^L |\Phi|^2 \, dz}{2 \int_0^L \Phi_+ \Phi_- \, dz} \right|^2.$$  \hspace{1cm} (8)

If this factor did not vary with time, as in common single-section lasers, it could be reduced in the photon rate equation and played no role for the dynamics. In our devices with individually changing carrier densities in different sections, however, the longitudinal mode shape and, hence, $K_z$ may exhibit considerable variations. In [15], it could be proved and demonstrated by a simple example that (2) is the correct photon rate equation, in these cases. The results of the present paper will point out the essential role of this factor $K_z$ for the DQS self-pulsations.

III. SELF-PULSATIONS IN A THREE-SECTION DFB LASER

As shown experimentally [4], [14], the SP’s can be obtained reproducibly in three-section lasers with two DFB sections and one integrated phase tuning section. It is the aim of this section, to demonstrate that the model can reproduce these SP’s when applied to the same device under the same conditions as in [4], [14].

In order to describe the device as realistic as possible we firstly extracted basic dependencies and parameters from measurements. Unfortunately, it was not possible to determine carrier densities with a sufficient accuracy. Therefore, we worked with the commonly used value $1 \cdot 10^{18}$ cm$^{-3}$ for the gain transparency concentration of bulk 1.55-$\mu$m InGaAsP–InP and $K_z$ for the DQS self-pulsations.
confined ourselves to the linear recombination model of (1). All remaining model parameters given in Table I could be determined from measurements. The coupling coefficient $\kappa$ was derived from the stopband of a single DFB-section. For a comprehensive extraction of other parameters, we used also single-section FP-lasers, processed on the same chip together with the DFB-lasers. Such characteristics as transparency current, differential gain, carrier induced wavelength shift or current-induced heating can be assumed to be nearly identical in both, the FP and DFB sections, but are much easier to extract from FP- than DFB-sections. From the independently measured gain and waveguide transparency currents, we deduced the mean recombination life time $\tau$ and the background optical losses $\alpha_0$. The gain-current measurements yielded the differential gain parameter $g'$ and convinced us furthermore, that the logarithmic gain model

$$g_k = g' n_t \ln \left( \frac{N_k}{V_k n_t} \right)$$

(9)

should be applied rather than the linear one used in [10] and [13]. This allows us to describe the DFB-sections with the same parameter set under very different bias conditions, i.e., (9) holds under both, low and high carrier injection in contrast to a linearized gain model. Up to now, we could not determine the effect of nonlinear gain saturation quantitatively. Therefore, it was not included in the model.

With this gain formula, the resulting model for the complex propagation constant in the DFB sections $\kappa = r, g$ can be written as

$$\beta_k = \frac{\pi}{\Lambda} + \kappa \frac{I_k}{L_k} - \beta_0 \left( \frac{N_k}{V_k} - n_t \right) + \frac{i}{2} \left( g_k - \alpha_0 \right).$$

(10)

The real parameters $\beta_1, \beta_2$ have been determined from the current induced wavelength shifts of isolated FP-sections. Above threshold, the carrier density is clamped, in these samples, and the measured red shift yields immediately $\beta_1$. Below threshold, the observed blue shift, in combination with the recombination law (1), yields $\beta_2$.

The observed current induced red shift is known to be due to the heating of the devices. In the self-pulsating DFB lasers, the injection levels of the individual sections are very different.

Thus, the correspondingly different heat production should cause a temperature difference. Using the figures measured with the FP lasers also for the multisection device, one estimates about 20–25 K difference between the two DFB sections in the regime of self-pulsations. However, the heat flow between the sections, which not appears when measuring isolated FP-sections, was expected to reduce this value. To clarify this point, we have estimated the longitudinal temperature distribution in our devices solving an approximate effective one-dimensional (1-D) heat flow equation. The results show that the heat flow between different sections plays a minor role. In the major part of each section, the active zone temperature has a constant value in good agreement with the measurements of isolated sections. The thermal transition region between two sections has an extend of only about 10 $\mu$m, which is small compared to the lengths of the sections. This rough estimate was confirmed by a comprehensive numerical calculation of the three-dimensional (3-D) temperature distribution [17]. In the following calculations, we neglect the effects of the transition region and assume a constant temperature within each section.

The resulting set of modeling data for the DFB-sections is collected in Table I. The model used for the phase tuning section will be given in the next Section.

Using this set of parameters, the model reproduces many experimentally observed features. For every combination of DFB-currents which yielded self-pulsations in the experiment, the model gave also self-pulsations after appropriately choosing the phase tuning current. This influence of the phase tuning current will be discussed in the next Section.

Here, the situation in a typical point of operation will be analyzed. The DFB currents are set to 133 mA and 8 mA, respectively. First, we have determined the stationary solution of (1) and (2) belonging to a stationary solution of the rate equations for operating conditions that show self-pulsations in the experiment.
relative to that of the reflector section, which means that the thermal detuning nearly compensates the electronic detuning. As a consequence, the long-wavelength stopband mode gets more feedback from the reflector section. It becomes the lasing mode, which is in agreement with the measurements. This conclusion is different from our earlier modeling [10], [13], where the neglect of the thermal detuning yielded lasing at the red flank of the first side lobe on the blue side of the reflector stop band.

As a next step, we applied the full dynamic model and found that the stationary solution becomes unstable and runs into a self-pulsation that is depicted in Fig. 3. The calculated repetition frequency of 10.6 GHz compares very well with the measured 10.5 GHz. The carriers in the reflector section are close to the transparency and do not participate in the self-pulsation. This supports our suggestion on this DFB section, being a more or less passive, but strongly dispersive reflector. The carriers in the gain section, on the other hand, are modulated by their interaction with the optical pulses. These features are the same as in our earlier modeling [10], [13], although now the thermal detuning guided us to a new operation point.

Concluding this section, it could be demonstrated, that the model reproduces self-pulsations of the same single stop band mode with the same repetition rate under the same conditions as in the experiment.

IV. ROLE OF THE PHASE SECTION

The phase section plays an important role for achieving controllable self-pulsations [4]. In this section, we shall present more details how this new element has been included into our single-mode model. Furthermore, we shall demonstrate that the extended model gives a good quantitative description of the main effects of this section.

The gap wavelength of the wave guide in the phase section is 1.3 \( \mu \)m. This is much shorter than the 1.57-\( \mu \)m operation wavelength. Thus, no interband transitions take place and the carrier density is not affected by the photons. It remains at a temporally constant value, which can be tuned by changing the injection current density \( I_p/L_p \). This tuning is accompanied by changes of both the effective refractive index and the free carrier absorption of the wave guide. We have measured these induced index and absorption changes for single FP sections with different lengths (200, 400 and 600 \( \mu \)m). The absorption measurements were based on the method of Hakki and Paoli, whereas the induced effective refractive index was obtained from the wavelength shift of the FP-modes. Representative results are collected in Fig. 4. The induced absorption \( \alpha_{\text{ind}} \) can be fitted by the third-order polynomial

\[
\frac{I_p}{L_p} = A\alpha_{\text{ind}} + B\alpha_{\text{ind}}^2 + C\alpha_{\text{ind}}^3
\]

(11)

with \( A = 18 \text{ mA} \), \( B = 0.28 \text{ mA cm}^{-1} \), \( C = 9.8 \cdot 10^{-3} \text{ mA cm}^{-2} \) (full line). This corresponds well to a free carrier absorption proportional to the carrier density combined with a superlinear recombination law. The measured index change can be sufficiently fitted by a linear relation to the induced absorption (dashed line). The measured index and absorption changes have been included into the model by using the expression

\[
\beta_p = \beta_{\text{op}} + \frac{i\alpha_{\text{op}}}{2} + (i - 2S\delta)\frac{\alpha_{\text{ind}}}{2}
\]

(12)

for the complex propagation constant of the phase section.

Using these formulas and the above parameter set, we have calculated the influence of the phase current \( I_p \) on the emission wavelength of the three-section device. The results for fixed DFB-currents (140 and 8 mA) are plotted in Fig. 5. A periodical variation of the wavelength with \( \phi \) can be observed, which agrees very well with the measured data. This agreement is achieved without fit parameters except the initial phase shift \( \beta_{\text{op}} \) which determines the position of the first maximum and the background absorption for which the reasonable value \( \alpha_{\text{op}} = 17 \text{ cm}^{-1} \) fits the amplitude of the first wavelength oscillation. The decreasing tuning efficiency with increasing phase current is due to the increasing free carrier absorption. The period of the wavelength variations increases because of the sublinear index-current relation. At large currents, deviations between theory and experiment occur because the analytic fit deviates somewhat from the measured effective index (cf. Fig. 4).

The measured wavelength tuning by the phase current is closely correlated with the observed regions of self-pulsations, which are highlighted in Fig. 5 by shadowed areas above...
the wavelength curves. During every wavelength period, the SP appear in a small region of $I_p$ immediately before the respective wavelength minimum. The calculated SP-regions (grey areas below the curves) have the same position relative to the theoretical wavelength minima. The width of the SP-regions is somewhat overestimated by the theory. It should be mentioned here, that hysteresis between stationary and SP states has been observed under certain conditions both in theory and experiment. A detailed analysis of this effect is beyond the scope of this paper, only SP-regions free of this hysteresis are depicted in Fig. 5.

Summarizing, the modeling results indeed show, that controlling the round-trip phase is deciding for reproducibly switching on and off the SP’s. The model satisfactorily describes the phase tuning effect and agrees very well with the experiment. Only the widths of the SP-regions are partially overestimated by the theory.

V. FURTHER COMPARISON THEORY-EXPERIMENT

In this section, we further test the quality of our model. It is investigated, how characteristic properties of the SP’s are described by the model, and how they fit quantitatively to the experiments.

A. DFB-Current Regions of DQS

Using the model with the above parameter set we have searched for SP-regions in the plane of the DFB-currents. The result is depicted in Fig. 6 for various phase currents. The location of the measured SP-regions compares qualitatively good with the theory. The typical bias conditions for DQS in theory and experiment are the same: One DFB-section is highly injected and can be viewed as gain section, the other section is near its gain transparency and acts like a dispersive reflector. Also, the evolution of the DQS-regions with readjusted phase current is the same in both, theory and experiment: they shift toward lower gain currents when the phase current increases (Fig. 6).

Again, the model tends to overestimate the extension of the SP-regions. This is probably due to the neglect of other limiting effects as, e.g., hole burning and nonlinear gain saturation. A further possible reason will be discussed in connection with a stability analysis in the next section.

B. Pulsation Frequencies

A typical indicator for the relevant SP-mechanism is the repetition frequency [13]. Using the model we have analyzed the repetition frequencies in dependence on the currents. Together with the measured values, the modeling results are shown in Fig. 7. On the left hand side of Fig. 7 the repetition frequencies are depicted in dependence on the current in the gain section. As visible, the calculated large-signal frequency agrees with the experiment. Deviations are less than 3%, which is quite surprising when applying such a simple model. The calculated SP-region according to the fixed phase and reflector current extends from $I_p = 120$ mA to roughly $I_p = 140$ mA, thereby exceeding the measured range by a factor of 2. At the borders the large signal frequency approaches the small-signal frequency according to the decreasing pulsation amplitude. In the middle of the region the deviations between them grow, but still the small-signal frequency gives an idea of the large signal frequency. As expected from the small-signal consideration the repetition frequency goes up with the lasing current, which is mainly due to the increased stimulated recombination (or photon-number) within this section.

On the right-hand side of Fig. 7, the evolution of the calculated frequencies for varying reflector current is compared
with measurements. As before, a surprisingly good agreement between measured and calculated large signal frequencies is observed. The tendency is the same and the deviations are less than 6%. The SP-region starts roughly at $I_r = 8$ mA in both, theory and experiment. At this point of small SP-amplitude the small-signal frequency (dashed line in Fig. 7) is close to the large signal frequency. With decreasing reflector current the deviations become quite large, which indicates an increasing SP-amplitude, so that small-signal considerations become not applicable. As mentioned in the beginning of this Section the extend of the SP-region is overestimated by the theory, which predicts SP’s over roughly the whole current range depicted in Fig. 7, in the experiment the laser switches to a stationary state below $I_r \approx 2$ mA.

C. Optical Spectra

Comparing the calculated and measured optical spectra in Fig. 8, again good agreement between theory and experiment is observed. With decreasing reflector current, the center line in the measured spectra (on the left of Fig. 8) decreases in height relatively to it’s satellites. This process is combined with a spectral broadening. The same thing is observed in the modeled spectra on the right of Fig. 8. This effect can be addressed again to the increasing SP-amplitude when decreasing $I_r$, as mentioned above. The increased amplitude modulation is accompanied by an increased chirp, which diminishes the value of the center line [14].

Concluding this section, it has been demonstrated, that the single-mode model for DQS extended by inclusion of a phase section, describes truly the basic features of the measured SP’s. This confirms the validity of the model and convinces us, that the observed SP’s are really due to the DQS-mechanism. The model describes qualitatively all measured dependencies and is with many respects also quantitatively right.

VI. THE MECHANISMS OF DISPERSIVE Q-SWITCHING

The validity of the single-mode model for describing the DQS has been demonstrated in the previous two sections. Now, we apply the model in order to get more insight into the

physical process of DQS. We shall clarify which mechanisms prevent a decay of the pulsations and keep them running.

A. Characteristic Marks of Q-Switching

The calculated temporal evolutions of important quantities during a representative cycle of a self-pulsation are depicted in Fig. 9. As is clearly visible, the losses $\gamma$ oscillate in antiphase to the gain $G$. This behavior is typical for the $Q$-switch operation [18]. In contrast to the usual case of saturable absorbers, however, it is the outcoupling losses of the device which oscillate in our case. More specific, the outcoupling through the reflector section varies due to the oscillations of the resonator wavelength, which are shown in the lower part of Fig. 9. Accordingly the $Q$-switching is of the dispersive type.

B. The Driving Force of the Self-Pulsation

Now we ask why the pulsations do not decay but are self-sustaining. For this discussion, it is useful to rewrite the photon rate equation (2) in the form

$$\frac{d}{dt} \ln S = G - \gamma + \frac{d}{dt} \ln \sqrt{K_2}.$$  

(13)

Here, the influence of the $K_2$-variation appears as a correction to the balance between the net gain $G$ and the outcoupling losses $\gamma$. Fig. 9 indicates that this correction has the same order of magnitude as the other two terms. It is negative when the photon number goes through the minimum in the time interval between $a$ and $b$. Thus, it tends to deepen this minimum. On the other hand, it is positive and enhances the maximum of $S$ during the time interval between $b$ and $c$. From this point of view, the changes of $\sqrt{K_2}$ play the role of a periodic driving force which tends to enhance the pulsation amplitude.
was artificially kept constant, however, between its internal net gain and its outcoupling. This correction has the
but is shifted forward by a quarter period. of the
from all other modes. -term in (13) represents the contribution of just
are in general a superposition during S ow e
and (which only is taken into account, on the other hand, has no influence on the -switching takes place, it is
-quantity with keeping \(K_z\) artificially constant. Circles: analytic approximation (14). For orientation, the power reflectivity of the reflector section within the wavelength range is drawn below.

The latter conclusion is consolidated by the small-signal stability analysis of the rate equations (1) and (2). Fig. 10 shows the calculated decay rate of a small perturbation of the stationary solution. The grey region indicates where this rate becomes negative. A perturbation in this region grows up into a self-pulsation. If \(K_z\) was artificially kept constant, however, the decay rate remained always positive (thin dashed line), i.e., a perturbation did never evolve into an SP.

Further insight can be gained if one benefits from the fact that the reflector section operates very close to its gain transparency. In this case, the carrier–photon coupling of this section is very small. Neglecting it completely yields the analytic formulas

\[
2\gamma_r = \left( \frac{N}{7} + v_g g \Gamma S \right)_N \left( \ln \sqrt{K_z} \right)_N \frac{v_g g \Gamma S}{N} \quad (14)
\]

\[
\omega_r = \sqrt{(G - \gamma)_N \cdot v_g g \Gamma S} \quad (15)
\]

for the damping rate of relaxation oscillations and the small-signal resonance frequency, respectively. Here, \(g\) and \(\Gamma\) denote the stationary gain coefficient and the longitudinal fill factor of the gain section, respectively, whereas the index \(N\) abbreviates the derivative with respect to the number of carriers in the gain section. The example drawn in Fig. 10 for the decay rate shows that these analytic formulae are a very good approximation which can be used for further conclusions.

The first term in the decay rate (14) represents the derivative of the total recombination rate of the gain section with respect to the carrier number in this section. This inverse change of the power emitted at the facets is the main physical effect which in our device keeps the pulsations running. As a consequence, the outcoupling losses respond with a certain delay to the variations of the carrier density. However, Fig. 9 shows that the modal outcoupling losses \(\gamma\) are not delayed with respect to \(G\). The situation changes, if we regard the \(K_z\)-term in (13) as a correction to \(G - \gamma\). This correction has the same period as \(G - \gamma\) but is shifted forward by a quarter period. Thus, the corrected losses experienced a phase shift forward, which is the same as a temporal delay. So we conclude that the delay between changes of the internally generated power and the resulting changes of the power emitted at the facets is the main physical effect which in our device keeps the pulsations running.

\[
D. \quad \text{The Role of Higher Modes}
\]

Let us finally discuss the same topic from the point of view of a mode decomposition of the optical field. The exact field amplitudes \(\Psi\) are in general a superposition \(\Psi = \Phi + \delta\) of the instantaneous lasing mode \(\Phi\) (which only is taken into account by our model) and of the contributions \(\delta\) from all other modes.

C. \quad \text{A More Physical Interpretation}

So far, we have demonstrated that the changes of \(K_z\) during a pulsation make the DQS-pulsations self-sustaining. Now we ask for the physical effects behind the corresponding abstract mathematical term in the photon rate equation.

At first glance, the relative change of the number \(S\) of photons in the laser mode should be completely given by the balance \(G - \gamma\) between its internal net gain and its outcoupling losses. However, the mode field \(\Phi\) follows immediately any change of the carriers, whereas the exact field needs a small but finite time to redistribute from the old mode shape to the new one. During the small redistribution time the relation between the internally generated power and the outcoupled power changes smoothly from the old mode value to the new mode value. Accordingly, the relative change of the photon number is different from \(G - \gamma\). In [15], it has been shown that the \(K_z\)-term in (13) represents the contribution of just this field redistribution effect to the number of photons in the laser mode.

The field redistribution in our particular device is dominated by the large distance of the facets from the gain section. As a consequence, the outcoupling losses respond with a certain delay to the variations of the carrier density. However, Fig. 9 shows that the modal outcoupling losses \(\gamma\) are not delayed with respect to \(G\). The situation changes, if we regard the \(K_z\)-term in (13) as a correction to \(G - \gamma\). This correction has the same period as \(G - \gamma\) but is shifted forward by a quarter period. Thus, the corrected losses experienced a phase shift forward, which is the same as a temporal delay. So we conclude that the delay between changes of the internally generated power and the resulting changes of the power emitted at the facets is the main physical effect which in our device keeps the pulsations running.
The corresponding exact photon number is

\[
\langle \Psi, \Psi \rangle = S + 2\Re(\Phi, \delta) + (\delta, \delta)
\] (16)

where \((\varphi, \psi)\) denotes the Hilbert space scalar product, i.e., the integral of \(\varphi^* \psi\). \(S = (\Phi, \Phi)\) is the photon number in the laser mode which evolves according to the single-mode rate (2). The second term reflects interferences between the lasing mode and higher modes. It is nonzero because optical modes of open resonators are not power orthogonal (see, e.g., the discussions in [11], [19], [20]).

It is useful to imagine the continuously varying carrier densities \(N_k(t)\) as a sequence of small steps. Suppose that just before one of these steps the higher order mode contribution \(\delta\) was negligible such that the exact photon number was well approximated by the single-mode contribution, say \(S_c\).

The exact field remains unchanged during the jump of the carrier densities and the exact photon numbers do not change as well. The modes however follow the carriers instantaneously. This has consequences. First, according to our single-mode rate (2), the photon number in the laser mode jumps to a new value \(S_{\delta}\), such that \(\sqrt{S_{\delta}/K_c}\) is kept constant. Second, the optical field contains after the jump a higher mode contribution \(\delta\), although it did not change and was single-mode before. Accordingly, the two last terms in (16) do not longer vanish. They compensate the jump of \(S\) and keep the exact photon number unchanged.

Now we regard the subsequent stage of constant carrier numbers. During this time, the amplitude of every mode grows or falls according to its own gain-loss balance. Thus, the higher mode contribution \(\delta\) decays relative to \(\Phi\). After some time only the lasing mode has survived, i.e., the field redistribution from the old to the new lasing mode has finished. Therefore, the redistribution time is mostly determined by the inverse gain margin. If it is short enough, the decay of the higher mode contribution appears as a jump of the exact photon number (16) from \(S_c\) to \(S_{\delta}\). From this point of view, the \(K_c\)-contribution to the photon rate equation considers the effect of the field redistribution in the limit of a vanishing redistribution time. In practice, this limit can be regarded as achieved if the inverse gain margin is short compared with the other characteristic times of the photon–carrier system. This is the same condition as for the single-mode approximation, which was well fulfilled for the cases considered in this paper.

VII. CONCLUSION

The single-mode model of dispersive self-\(Q\)-switching has been extended to lasers composed of two DFB-sections and one phase tuning section. The model was applied to a specific structure using experimentally extracted model parameters. Very good agreement with measured self-pulsations was obtained concerning the role of the phase tuning section, the regions of SP’s in the plane of the two DFB-currents, the repetition frequencies as well as the evolution of the optical spectra. These results confirm the dispersive self-\(Q\)-switching as the basic mechanism for the self-pulsations observed in these devices. Furthermore, the modeling has shown that the delay between changes of the internal generation of power and the corresponding changes of the power emitted at the facets, which enters the theory via variations of the longitudinal excess factor of spontaneous emission, plays an important role for keeping the pulsations running. Finally, the conclusion can be drawn that the single-mode model is a good basis for further improvement and optimization of the self-pulsating lasers.

ACKNOWLEDGMENT

The authors want to thank Z. Zhu and J. Hörer-Dragendorf for measurements on the phase tuning section, and S. Reichenbacher for measurements on the \(\alpha\)-factor and on current induced heating. Furthermore, they also thank J. Piprek of the University of Delaware, for the 3-D calculations of temperature distributions.

REFERENCES


Uwe Bandelow was born in Templin, Germany, in 1962. He received the Diploma degree in 1991 and the Doctoral degree in 1994, both in physics, from Humboldt University, Berlin.
Since 1991, he has been involved in research projects on quasi-3-D modeling and on the dynamics in multielectrode DFB lasers at Humboldt University and at the WIAS Berlin. His present research interests concentrate on the dynamics in semiconductor lasers.

Hans-Jürgen Wünsche (M’93) was born in Nossen, Germany, in 1948. He received the Diploma, the Dr. rer. nat, and the Dr. sc. nat. degrees in physics from Humboldt University, Berlin, in 1972, 1975, and 1983, respectively.
He is still with the Humboldt University, has carried theoretical research on tunneling and high excitation phenomena in semiconductors. His current research interests concentrate on the theory of photonic devices.

Bernd Sartorius was born in Bad Soden, Germany, in 1948. He received the Diploma in physics in 1975 and the Ph.D. degree in 1982, both from the Technical University, Berlin. In 1982, he joined the Heinrich-Hertz-Institut, where he first was engaged in nondestructive characterization of III–V semiconductor materials and processing. In 1990 he became head of a group developing InGaAsP–InP lasers and amplifiers. His current research interest is directed toward all-optical signal processing using dispersive effects in novel multisection DFB lasers.
Dr. Sartorius is member of the German Physical Society.

Martin Möhle was born in Freudenstadt, Germany, in 1962. He received the Diploma degree in physics from the University of Stuttgart in 1988, and the Ph.D. degree from the Technical University of Berlin in 1992.
In 1988, he joined the Heinrich-Hertz-Institut, Berlin, where he is engaged in research, development and fabrication of semiconductor lasers, optical amplifiers, and optoelectronic circuits.