Mechanisms of Fast Self Pulsations in Two-Section DFB Lasers

Hans Wenzel, Uwe Bandelow, Hans-Jürgen Wünsche, and Joachim Rehberg

Abstract—We show theoretically, that the detuning between the resonance frequencies of differently pumped DFB sections gives rise to two different pulsation mechanisms, 1) dispersive self Q-switching of a single-mode and 2) beating oscillations between two modes of nearly equal threshold gain. Our analysis is based on the dynamic coupled wave equations accomplished with carrier rate equations. We demonstrate the existence of certain isolated values of the detuning between both sections, at which two longitudinal eigenmodes become degenerate. In the degeneration point, the longitudinal excess factor of spontaneous emission has a singularity and the system of eigenmodes becomes incomplete. We derive reduced equations governing the dynamics in the vicinity of degeneration points. For an example device, the numerical integration of these equations clearly demonstrates the two different self pulsations with repetition rates of more than 100 GHz.

I. INTRODUCTION

SELF-PULSATING distributed feedback (DFB) semiconductor lasers are promising as optical sources with electrically tunable repetition rates (13-80 GHz yet demonstrated [1], [2]) combined with a locking function to an optically injected modulation. Clock recovery with such devices can play an important role in future high speed telecommunications systems. All-optical clock recovery at 18 Gb/s has already been demonstrated [3].

The nature of these fast self pulsations is not yet completely understood. As has already been pointed out by Moehrle et al. [1], they must be due to a mechanism specific for DFB lasers and cannot be attributed to the saturable absorber mechanism, which is responsible for the most self pulsations in Fabry–Perot lasers. Syvridis et al. draw the same conclusion for the self pulsations they observed in a three section DBR laser [4], which is also supported by the recent results of Duan et al. [5]. To our knowledge, two different pulsation mechanisms specific for DFB lasers have been discussed up to now.

The first one is related to an instability induced by spatial hole burning, which has been found by Schatz [6]. According to Schatz, above a critical injection level, the longitudinal carrier distribution of a single-section DFB laser becomes unstable against symmetry breaking spatial perturbations. It was argued by Phelan et al. [7], that the self pulsations which they detected in three-section DFB lasers, could be due to an instability of this type. By a numerical simulation of a single-section DFB laser with the transmission line model, Lowery has demonstrated [8], that the Schatz instability can lead to self pulsations. This was also found by Marcenac and Carol [9] for two section devices. They have shown with the help of a comprehensive time domain model (TDM) that SP of this type appear in the nearly symmetric pump regime \( I_1 \approx I_2 \). The repetition rates of these self pulsations, however, are comparable 'slow' due to the long build up time of the spatial hole. Therefore, we do not believe that the self pulsations with some tens of GHz reported in [1]-[3] can be caused by this spatial hole burning mechanism.

The second pulsation mechanism is specific for multisection DFB lasers and can be regarded as a dispersive self Q-switching. It was first proposed and analyzed by some of the authors [10] on base of a simple single-mode model and has meanwhile been confirmed with the time domain model of Marcenac and Carol [9], [11]. It takes effect, if one section is driven at a high injection level as the "lasing" section, whereas the other one operates near the waveguide transparency and acts mainly as a wavelength dependent reflector. The position of the lasing wavelength relative to the feedback spectrum of the reflector section depends on the refractive index difference between both sections, which changes if the carrier densities varies. If it is close to a feedback minimum of the reflector section, its shift due to any density fluctuation causes a change of the feedback from the reflector section, i.e., a change of the resonator's \( Q \)-value. This dispersive switching of \( Q \) can act as the driving force for a self sustaining pulsation in a similar manner as the switching of the resonator's \( Q \) by a saturable absorber in FP devices. In case of small and deep feedback minima, already small carrier density variations cause remarkable changes of the feedback, which results in rather high-repetition frequencies.

In this paper, we present a deeper theoretical investigation of the stationary and dynamical properties of two-section DFB lasers. We start in Section II with an expansion of the dynamic coupled wave equations in terms of the instantaneous modes of the compound resonator, which gives a general frame for the more intuitive single-mode description in [10]. The compound cavity modes are studied in more detail in Section III for a simple model neglecting effects of spatial hole burning. We detect the existence of singular points in the plane of the complex detuning parameter (Bragg frequency
and gain difference between both sections), where two modes become completely degenerate. It is shown, that the regions of “dispersive self Q-switching” are located close to these degeneration points. Furthermore, we find there regions, where the two nearly degenerate modes have slightly different frequencies but an equal threshold gain, which should enable beating oscillations. The phenomenon of mode degeneration is accompanied by some strange features as, e.g., a singularity of the longitudinal excess factor of spontaneous emission. In this context, we have performed a mathematical examination of the coupled-mode equations, the main results of which are collected in the Appendix. Among others, it has been found, that the mode expansion fails in the degeneration points due to the incompleteness of the system of eigenmodes. We can, however, provide a scheme for constructing a complete basis. On this base, a modified set of dynamic equations is obtained in Section IV for the most interesting case of two degenerate modes. The numerical solution for an example device is presented for two different regimes of injection in Section V. In the first case, the “dispersive self Q-switching” type is reproduced, establishing our considerations in [10]. In the second case, another type of self oscillations appears, which is due to mode beating with oscillation frequencies of the order of 100 GHz. The relation of this pulsation type to similar theoretical and experimental observations in broad single stripe or twin stripe lasers [13], [14], in short external cavity lasers [15], [16], and two section DFB lasers [2] is briefly discussed.

II. MODE EXPANSION OF THE DYNAMIC COUPLED WAVE EQUATIONS

There are several possibilities of modeling the temporal behavior of DFB lasers. Existing dynamic models are based, for example, on the solution of the dynamic coupled wave equations [9], [17], on the transfer-matrix method [18], on the transmission-line method [19] or on the power-matrix method [20]. All methods should be equivalent and should yield similar results.

We have chosen as starting point of our investigations the dynamic coupled wave equations. After averaging over the transverse plane and separating terms varying rapidly in space and time, the slowly varying amplitudes of forward and backward travelling waves, \( \Psi^+(z,t) \) and \( \Psi^-(z,t) \), respectively, fulfill the equations [21]

\[
-i \frac{\partial \Psi(z,t)}{\partial t} = H \Psi(z,t),
\]

where we have introduced the vector notation

\[
\Psi = \begin{pmatrix} \Psi^+ \\ \Psi^- \end{pmatrix},
\]

and \( H \) is a matrix operator given by

\[
H = i \sigma_z v_g \frac{\partial}{\partial z} - M(z,t)
\]

with

\[
\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
\]

and

\[
M(z,t) = v_g \left( \begin{array}{cc} \beta(z,t) - \kappa^- & \kappa^+ \\ \kappa^- & \beta(z,t) - \kappa^+ \end{array} \right).
\]

In (4), \( \beta \) is the propagation constant at a fixed central wavelength. It depends on the longitudinal position \( z \) and on time \( t \) via the carrier density \( N(z,t) \). \( \Lambda \) is the Bragg grating period. The nondiagonal elements \( \kappa^{\pm} \) of \( M \) cause a coupling of the amplitudes of the forward and backward travelling waves \( \Psi^+ \) and \( \Psi^- \) in the laser cavity due to the Bragg grating. In this paper, we will neglect spontaneous emission which would otherwise occur as an additional stochastic driving term in (1). This is possible because above laser threshold the spontaneous emission into the lasing mode is much smaller than the stimulated emission and we expect that spontaneous emission has only a small effect on self pulsing as it was found in two-section FP lasers containing a gain and a saturable absorber section [22] and which has been confirmed with a time domain model [11]. At the facets of the laser we assume reflecting boundary conditions for the travelling wave amplitudes,

\[
\Psi^+(0,t) = r_0 \Psi^-(0,t) \quad \text{and} \quad \Psi^-(L,t) = r_L \Psi^+(L,t),
\]

with the amplitude facet reflectivities \( r_0 \) and \( r_L \). Equation (1) has to be supplemented by an initial condition and by a time-evolution equation for the carrier density \( N \), which will be given later. Due to the stimulated recombination, the carrier density depends itself on \( |\Psi|^2 \) and we shall end up with a nonlinear problem.

Our approach to (1) applies the general mode expansion technique outlined in [23]. It is based on the expansion

\[
\Psi(z,t) = \sum_m f_m(t) \Phi_m(z,t),
\]

in terms of the instantaneous modes \( \Phi_m \), which are the solutions of the quasistationary coupled-mode equations

\[
H \Phi_m(z,t) = \Omega_m(t) \Phi_m(z,t)
\]

subject to the boundary conditions (5). These equations depend parametrically on time. The complex mode frequencies \( \Omega_m \) are the eigenvalues of \( H \) to be determined. The real part of \( \Omega_m \) is the modal optical frequency (relative to a reference value), the imaginary part is the decay rate of mode \( m \) and proportional to its threshold gain.

For the following, we define the inner product

\[
\langle \Phi, \Psi \rangle \equiv \int_0^L (\Phi^- \Psi^+ + \Phi^+ \Psi^-) \, dz.
\]

Note, that this inner product is different from the Hilbert space scalar product

\[
(\Phi, \Psi) \equiv \int_0^L (\Phi^{**} \Psi^+ + \Phi^{*} \Psi^-) \, dz.
\]

As is well known (cf. [24, p. 41 and references therein]), the quotient of both products gives the so called longitudinal
excess factor of spontaneous emission into a mode \( m \) by

\[
K_2 = \frac{|\langle \Phi_m, \Phi_m \rangle |^2}{|\langle \Phi_m, \Phi_n \rangle |^2} .
\]  

(10)

The operator \( H \) is symmetric in the sense of (8), i.e.,

\[
\langle H \Phi, \Psi \rangle = \langle \Phi, H \Psi \rangle
\]

(11)

holds for twocomponent functions \( \Phi \) and \( \Psi \), which obey the boundary conditions (5). Hence, any two of its eigenfunctions belonging to different eigenvalues are orthogonal,

\[
\langle \Phi_m, \Phi_n \rangle = 0 \quad \text{for} \quad \Omega_m \neq \Omega_n .
\]  

(12)

This is a generalization of the orthogonality relation reported in [25] to be fulfilled by the longitudinal modes of Fabry–Perot lasers. It can also be regarded as the Hilbert space orthogonality between modes and adjoint modes (see, e.g., the discussion in [26]). Note, that orthogonality holds for arbitrary functions \( \beta(z) \). Assuming \( \langle \Phi_m, \Phi_m \rangle \neq 0 \), it is always possible to normalize the modes according to

\[
\langle \Phi_m, \Phi_m \rangle = 1 .
\]  

(13)

Inserting (6) into (1) and taking into account (7), (12) and (13) we obtain the following dynamic multimode equations for the amplitude coefficients \( f_m \):

\[
\frac{df_m}{dt} = i\Omega_m f_m - \sum_{k \neq m} \frac{\langle \Phi_m, H \Phi_k \rangle}{\Omega_m - \Omega_k} f_k .
\]  

(14)

The summations in (6) and (14) extend over the infinite set of modes. However, only modes carrying sufficiently much optical power give a significant contribution. According to experimental observations [1], this is often only one single mode, say \( m = 0 \). Neglecting the side modes, the coupling term in (14) vanishes. This yields the single-mode equations utilized in [10]. Validity conditions for the single-mode approximation can be obtained if considering (14) for \( m \neq 0 \). Regarding \( f_0 \) as large, all terms \( k \neq 0 \) in the sum can be dropped. Each side mode is driven only by the main mode. Its amplitude remains small, if the carrier profile varies slowly in time or with a weak modulation amplitude and the gain margin \( 23 \% \) \( \Omega_m - \Omega_0 \) is large compared with the repetition frequency of the pulsation.

III. THE APPEARANCE OF DEGENERACY IN THE MODE SPECTRUM

In order to apply (14), one needs the complex mode frequencies \( \Omega_m \) and the associate mode functions \( \Phi_m \). Let us study these quantities by means of a simple but characteristic example. We consider a nonphase shifted two-section DFB laser with sectional lengths \( L_1 = L_2 = 300 \mu \text{m} \), a coupling coefficient of \( \kappa = 100 \text{ cm}^{-1} \) (corresponding to \( \kappa L = 6 \)) and having perfectly AR-coated facets. Neglecting spatial hole burning, we work with spatially constant values \( N_k \) for the carrier concentration in every section \( k = 1, 2 \) and hence with constant values \( \beta_k \). Under these conditions, it is useful to consider the dimensionless reduced mode frequencies

\[
\Delta_m = \int_0^L \left[ \beta(z) + \frac{\Omega_m}{\pi \Lambda} - \frac{\pi}{\Lambda} \right] dz .
\]  

(15)

These quantities depend on the \( \beta_k \) only via the detuning \( \delta = \beta_2 - \beta_1 \) \( L \) between the two sections (real part: index detuning, imaginary part: gain detuning).

Fig. 1 shows the evolution of the corresponding mode spectra along a straight line in the complex \( \delta \)-plane, which is drawn as a full line in the insert. The lasing mode is drawn thick solid. In the case of zero detuning (homogeneous pumping), one clearly recognizes the stop band in the real part of \( \Delta_m \). Equivalent modes at opposite sides of the stop band have equal threshold gain, in this case. With increasing detuning, the mode frequencies below the stop band move upwards, whereas those above the stop band move downwards, thereby diminishing the stop band width. The threshold gains show a rather complicated nonmonotonous behavior. At particular values of the detuning, the frequencies of modes from below and above cross each other. Three types of mode crossing occur in Fig. 1:

1) Mode crossing with frequency degeneration. This case, where the frequencies of both modes intersect and the threshold gains approach without touching each other is indicated by squares.

2) Mode crossing with gain degeneration, which is indicated by rhombi. Complementary to the former type, the threshold gains of both modes intersect, but not the frequencies.

3) Mode crossing with complete mode degeneration. Frequencies as well as threshold gains are crossing at the same time. The corresponding points are indicated by the circles.\(^1\)

Because in the first case the threshold gains of the other modes are much larger than the threshold gain of the lasing mode, we can apply a single-mode approximation which leads to the type of self pulsations investigated in our paper [10]. As visible, the threshold gain of the lasing mode has a local maximum. This corresponds to a minimum in the feedback spectrum of the less pumped section as discussed in [10]. Self pulsations due to dispersive self \( Q \)-switching would occur at the right flank of the “threshold gain hill.”

In case of a mode crossing with gain degeneration, there are two modes with different oscillation frequencies but identical threshold gains. Therefore, we expect a new type of self oscillations due to mode-beating (if the modes were the lasing ones). The corresponding beating frequency would be determined by the difference between the optical frequencies of both modes.

We have recognized that the third case of complete mode degeneration appears at several isolated points in the complex

\(^1\)In Fig. 1, there is also a third mode with the same threshold gain. We disregard this mode in the following and consider only the two crossing modes. Later we will see, that in the self-pulsating state the disturbing mode has a sufficient gain margin (cf. Fig. 2). We note, that by changing other parameters (section lengths, reflectivities and their phases), it could be shifted to higher threshold gains.
The rhombi gain degeneration and the circles a point of full mode degeneration. The renormalized complex mode frequencies \( A \), along a straight line (Fig. 1). Evolution of the imaginary (upper curves) and real (lower) parts of the complex plane of the detuning parameter \( \delta \). The squares denote frequency degeneration, the rhombi gain degeneration and the circles a point of full mode degeneration.

The insert shows furthermore the location of the degeneration points of the mode pairs \([+1,-1]\) (circle), \([+2,-1]\) (square), and \([+1,-2]\) (rhombus), the dynamic trace through the point of full degeneration of the mode pair \([+1,-1]\) (dashed line) and the regions of dispersive self-Q-switching and mode-beating pulsations as full circles labeled by (i) and (ii), respectively.

\( \delta \)-plane, each of them can be attributed to a certain pair of modes. The insert in Fig. 1 shows three of these points for our example. The full line along which the mode evolution is depicted in Fig. 1 has been chosen to meet exactly the degeneration point of the mode pair \([+1,-1]\). It bypasses the other degeneration points which is reflected in the spectra as the corresponding type of mode crossing. From this point of view, the two types of mode crossing with gain and frequency degeneration are only different manifestations of a nearby point of complete degeneration.

**IV. Dynamic Equations for Nearly Degenerate Modes**

In this section, we investigate what happens dynamically in the vicinity of a mode degeneration point. Unfortunately, the multimode equations (14) break down there, because the frequency difference in the denominator of the coupling term approaches zero. The perturbation theory of quantum mechanics for degenerate states, as described, e.g., in [27], is also not applicable, because the evolution operator \( \mathbf{H} \) defined by (3) is not self-adjoint. It is well known (see, e.g., [26]), that the nonselfadjointness of the optical equations can result in unusual and surprising physical properties of the optical modes as the appearance of the Petermann factor of excess spontaneous emission. To our knowledge, however, the implications of degenerate modes have not yet been regarded. Therefore, we have performed detailed mathematical investigations of our basic equations with respect to this phenomenon. A correct presentation of the findings and of the proofs needs a rather special mathematical language which not all readers of this journal will be familiar with. So we give a more heuristic description only of what is essential for our particular case of twofold degenerate modes. A more comprehensive but still brief formulation in form of mathematical theorems can be found in the Appendix. The detailed development of the material including proofs will be published in a mathematical journal [28]. Let us regard the point of degeneracy of any couple of modes, say \( m = 0 \) and \( m = 1 \) with the common eigenvalue \( \Omega_2 = \Omega_1 \). The other modes \( m > 1 \) are assumed to be not degenerate. We have found, that the two eigenfunctions \( \Phi_0 \) and \( \Phi_1 \) approach each other when \( \Omega_2 \rightarrow \Omega_1 \), and become identical in the point of degeneracy (Appendix, Theorem 1.1). This behavior is completely different to what is known from quantum mechanics, where the number of states does not change in case of degeneracy. It implies, that the dimension of the space spanned by our two eigenfunctions reduces from 2 to 1, i.e., one degree of freedom seems to disappear. This lost degree of freedom can be regained, however. It is spanned by a new function \( \Phi_1^{\prime} \), which can be determined from the single eigenfunction \( \Phi_0 \) by solving the inhomogeneous equation (Appendix, Theorem 1.2)

\[
[\mathbf{H} - \Omega_2] \Phi_1^{\prime} = \Phi_0.
\]  

This construction corresponds to the construction of Jordan's normal form of finite quadratic matrices and is also well known.

\(^3\)At the same time, they remain orthogonal in the sense of (12). As a consequence, the product \( \langle \Phi_0 , \Phi_0 \rangle \) vanishes in the degeneration point. It should be remarked, that this latter fact means that the longitudinal excess factor (10) of spontaneous emission into each of these modes diverges when approaching the degeneration point.
from linear differential equations with degenerate eigenvalues (see, e.g., [29]). Therefore, we shall call $\Phi_1'$ the (first) Jordan vector belonging to the degenerate eigenvalue $\Omega_0$. It should be remarked here, that instead of solving an inhomogeneous equation (16), the Jordan vector can also be calculated according to

$$\Phi_1' = \frac{\partial \Phi_0}{\partial \Omega_0}$$  

(17)

which can be readily shown by deriving (7) for $m = 0$ with respect to $\Omega_0$. The system of eigenfunctions becomes again complete, if this Jordan vector $\Phi_1'$ is put on the vacant position of $\Phi_1$ (Appendix, Theorem 2). As a consequence, the dynamics close to a degeneration point can be described by an expansion of the field amplitude with respect to this complete system. If we rename the Jordan vector by $\Phi_1$ (without prime), this expansion has the same form as (6), but now with time-independent eigenfunctions $\Phi_m$.

For the following, we assume a sufficiently large gain margin of the higher modes $m > 1$ and confine ourselves to the two leading terms

$$\Psi(z, t) = f_0(t)\Phi_0(z) + f_1(t)\Phi_1(z).$$  

(18)

Furthermore, we assume that $\Phi_0$ and $\Phi_1$ were made orthonormal according to the scalar product (9),

$$\langle \Phi_i, \Phi_j \rangle = \delta_{i,j} \quad \text{for } i,j = 0 \text{ or } 1.$$  

(19)

Inserting (18) into (1), multiplying the result from the left with $\Phi_0$ and $\Phi_1$, respectively, integrating over the whole laser length and taking into account (19), leads to the two equations

$$i \frac{df_0}{dt} + \Omega_0 f_0 + f_1 - \sum_{m=0,1} (\Phi_0, \Delta M \Phi_m) f_m = 0$$  

(20)

and

$$i \frac{df_1}{dt} + \Omega_1 f_1 - \sum_{m=0,1} (\Phi_1, \Delta M \Phi_m) f_m = 0,$$  

(21)

which describe the dynamics of two nearly degenerate modes. Note the appearance of the term $f_1$ in (20) which is due to (16). In (20) and (21), the diagonal matrix

$$\Delta M \equiv M - M_0 = \begin{pmatrix} \beta(z,t) - \beta_0(z) & 0 \\ 0 & \beta(z,t) - \beta_0(z) \end{pmatrix}$$  

(22)

gives the deviation of $M$ from the degeneration point.

Before finishing this section, let us consider the question, whether more than two modes can become degenerate in one and the same point. Principally, this is possible and the general formulation of the theorems in the Appendix indicates how to proceed in such a case. However, considering the eigenvalues $\Omega_m$ as functions of the problem parameters, $n$-fold degeneracy appears as a crossing of $n$ curves in one point (Appendix, Theorem 3). It is obvious, that such multiple crossings are much more seldom than the crossing of two curves. They can be disregarded, unless a special device design forces their appearance.

V. EXAMPLE CALCULATIONS

Before numerically integrating the equations (20) and (21) for the amplitudes $f_0$ and $f_1$, we have to specify how the dependence of $\beta$ on the carrier density looks like and how the carrier density temporarily evolves.

It is the aim of this paper to clarify theoretically which are the basic DFB-mechanisms behind the experimentally detected fast SP's. For these purposes, we keep our model as simple as possible and disregard such effects as higher modes, spatial hole burning, nonlinear gain suppression, gain levering, finite facet reflectivities, and thermally induced wavelength shifts, which in our opinion are not responsible for this phenomenon but can modify it. The role of these additional effects will be studied in forthcoming papers. A first direct comparison with a much more sophisticated model presented in [11] has already established the validity of the most simplifications. In this sense, we work in the following with spatially constant carrier densities $N_k$ and propagation constants $\beta_k(t)$ in each section $k$. This is a good approximation because in our case the longitudinal distribution of the carrier density is mainly determined by the different carrier injection levels of the two sections [11]. Further assuming the propagation constant to depend linearly on the carrier density and neglecting the nonlinear gain suppression, we use

$$\beta_k(t) = \frac{\pi}{\lambda} + (-1)^k \frac{\delta_\beta}{2} - \frac{i}{2} \alpha_0 + \frac{2\pi}{\lambda} n' N_k(t)$$

$$+ \frac{i}{2} g'[N_k(t) - N_{tr}], \quad k = 1, 2,$$  

(23)

where $n'$ is the differential refractive index, $g'$ is the differential gain, $N_{tr}$ is the transparency carrier density and $\alpha_0$ are the internal optical losses.

The unusual real parameter $\delta_\beta$ will play an important role. It represents a hypothetic static detuning between the Bragg gratings of the two sections. During the dynamic evolution, due to the variations of the carrier densities, the detuning parameter $\delta = (\beta_2 - \beta_1)L$ moves only along a straight line in the complex plane, whose slope is given by the Henry factor $\alpha_H = 4\pi n'/g'\lambda$ ($\approx -4$). This line, which we will call the dynamic trace, crosses the real axis at $\delta_x$. All published model calculations up to now were restricted to the case

$$\alpha_H \approx -4.$$
A. Dispersive Self Q-Switching Self Pulsations

First we chose the currents to be very different between both sections, for example \( I_1 = 87 \text{ mA} \) and \( I_2 = 42 \text{ mA} \). The temporal evolution of the two densities and optical powers is shown for this case in Fig. 3. The pulsation frequency is approximately 10 GHz and both powers oscillate with the same phase. The carrier density in the reflector section (in our example section 2) shows only very weak oscillations, because it is close to transparency and, therefore, couples only weakly to the optical field. The carrier density in the laser section is larger and shows more pronounced oscillations. The optical spectrum calculated by a fourier transformation of the complex amplitudes is depicted in Fig. 4. It is similar to that presented in [9] and demonstrates clearly the single-mode emission (note the logarithmic scale). The blue shift of about 8 nm with respect to \( \lambda_0 \) is due to the high carrier density in the laser section. A fine structure can be observed, which is closely related to the relatively strong higher harmonics of the amplitude modulation. Its spacing of about 0.08 nm corresponds well to the pulsation frequency of 10 GHz. The central fine structure line is reduced due to the chirp connected with the oscillation of the carrier density in the laser section. The side maxima correspond well to the spectral positions of the maxima of the reflectivity spectra \( R_2 \) of section 1 in the moments of minimum and maximum carrier density, which differ by about 0.3 nm. The first side minimum of the \( R_2 \)-spectrum lies within this spectral range. This proves, that the self pulsations plotted in Fig. 3 are due to dispersive self Q-switching in the sense of our previous paper [10]. That point in the complex plane of the detuning parameter \( \delta \), around which these pulsations appear, and the corresponding complex frequencies \( \Delta_m \) are indicated in the insert of Fig. 1 as well as in Fig. 2 by fat dots labeled with \((i)\). They are located very close to the degeneration point. We conclude that this type of self pulsations is closely correlated with the appearance of mode degeneracy.

From this point of view a change of device parameters (e.g., length's or \( \kappa \)) would enlarge the selfpulsation-region when the dynamic trace approaches a degeneration point, and diminish otherwise. For realistic currents the pulsation frequency itself does not exceed an upper limit mainly determined by the differential index \( n' \).

\[ \delta_s = 0. \] In this case, the dynamic trace passes relatively far from the degeneration points in our high \( \kappa L \) device, and no selfpulsation would appear. A nonvanishing static contribution, however, opens a new degree of freedom. It can be used to shift the dynamic trace closer to the degeneration points. In the following example, we choose \( \delta_s = 195.1 \text{ cm}^{-1} \), which together with the parameters in Table I ensures that the dynamical trace passes directly through the degeneration point, as illustrated by the dashed line in the insert of Fig. 1. Such a static detuning corresponds to a relative difference between the grating periods of both sections \( \Delta \lambda/\lambda = \Lambda \delta_s/2\pi \) of only about \( 8 \times 10^{-4} \), which can be well-aimed realized, for example, with the method of bent waveguides [30].

The dynamic equations for the carrier densities \( N_k \) read

\[
\frac{dN_k}{dt} = \frac{I_k}{eV_k} - \frac{N_k(t)}{\tau} - g' [N_k(t) - N_{sr}] \frac{\lambda}{hcV_k} \int_{(L_k)} |\Psi(z,t)|^2 dz, \\
k = 1, 2, \\
(24)
\]

with

\[
|\Psi|^2 = |f_0|^2 (|\Phi_0^+|^2 + |\Phi_0^-|^2) + |f_1|^2 (|\Phi_1^+|^2 + |\Phi_1^-|^2) + 2Re\{f_0* f_1(\Phi_0^+* \Phi_1^- + \Phi_0^-* \Phi_1^+)\}, \\
(25)
\]

and where \( I_k \) is the current into the \( k \)-th section, \( V_k \) its volume and \( \tau \) is the spontaneous carrier lifetime. Using the values of the parameters collected in Table I, a numerical integration of the model equations (20), (21) and (24) yields two different types of self pulsations in dependence of the chosen currents.

### Table I

VALUES OF THE PARAMETERS USED IN THE CALCULATIONS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Explanation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda ) [( \mu \text{m} )]</td>
<td>Central vacuum wavelength</td>
<td>1.55</td>
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<tr>
<td>( n_0 )</td>
<td>Facet reflectivity</td>
<td>( 10^{-4} )</td>
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<td>( \kappa ) [( \text{cm}^{-1} )]</td>
<td>Coupling coefficient ( x' = \kappa' )</td>
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<td>( L ) [( \mu \text{m} )]</td>
<td>Laser Length</td>
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<td>( n_g )</td>
<td>Effective group index</td>
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<td>( \theta' ) [( \text{cm}^2 )]</td>
<td>Differential refractive index</td>
<td>( -8 \times 10^{-23} )</td>
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<td>( \alpha_{r} ) [( \text{cm}^{-1} )]</td>
<td>Differential gain</td>
<td>( 1.6 \times 10^{-18} )</td>
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<td>( n_c ) [( \text{cm}^{-3} )]</td>
<td>Transparency carrier density</td>
<td>( 2.1 \times 10^{18} )</td>
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<td>( a_0 ) [( \text{cm}^{-1} )]</td>
<td>Internal absorption</td>
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</tr>
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<td>( \delta ) [( \text{cm}^{-1} )]</td>
<td>Static detuning</td>
<td>195.1</td>
</tr>
<tr>
<td>( V_k ) [( \mu \text{m}^2 )]</td>
<td>Sectional volume ( V_1 = V_2 )</td>
<td>121.9</td>
</tr>
<tr>
<td>( \tau ) [( \text{ns} )]</td>
<td>Carrier lifetime</td>
<td>1.25</td>
</tr>
</tbody>
</table>
Although only a small environment of the point (i) in the complex $s$-plane gives rise to dispersive self $Q$-switching SP's, they appear in an extended area of the $I_1-I_2$-plane (shadowed in Fig. 7). The current $I_2$ has to keep the waveguide of the reflector section close to its transparency, where the feedback minima are sufficiently deep. The corresponding transparency current is about 35 mA. With increasing $I_1$, the SP region shifts slightly toward higher $I_2$-values which compensates for the increasing stimulated recombination in the reflector section. No upper limit for the laser section current $I_1$ appears. If the two currents are interchanged, no SP region appears because the device is not symmetric due to the finite static detuning $\delta_0$.

**B. Mode Beating Self Pulsations**

If we diminish the difference in the currents, $I_1 = 87$ mA and $I_2 = 62$ mA, we obtain stable self oscillations with a very high frequency of 103 GHz shown in Fig. 5. As can be seen, the carrier densities in both sections are almost equal to each other. The point corresponding to their mean value is indicated in the insert of Fig. 1 as a fat point labeled by (ii). Fig. 2 shows that, within the approximation (20) and (21), the two modes $+1$ and $-1$ on the opposite sides of the stop band are gain degenerate. Thus, the pulsations shown in Fig. 5 represent the expected beating between these two modes. This conclusion is further supported by the agreement between the repetition frequency and the mode spacing (103 GHz) and by the calculated optical spectrum in Fig. 6, which clearly shows two lines with nearly equal intensity. In contrast to Fig. 4, no fine structure appears in the spectrum, because the higher harmonics and the chirp are negligible, in this case.

At first glance, it seems to be hardly possible to hold in practice the device exactly and stable on that singular point, where the two modes become gain degenerate. Surprisingly, the opposite is the case: this single point (ii) on the dynamic trace corresponds not to a likewise single point in the current-current plane of the device, but is mapped onto the rather extended area around the symmetry line $I_1 = I_2$, which is hatched in Fig. 7. The reason lies in the longitudinal spatial structure of the two beating modes. The calculated intensity distributions are concentrated within one section and exhibit a
mode beating to the face to face operation of two single-section DFB lasers. The shadowed area is the region lines enclose dotted lines give the relative power-current plane of the example device with this detuning, their stop bands are spectrally shifted relative to these lasers oscillates on that side of its stop band, which gets this point of view, each section plays a double role. It is the case. The nearly rectangular shape of the beating region in Fig. 7, as well as the straight lines of constant total power (full) and power ratio (dotted) within this region show that this mode beating are independent of $\kappa L$, i.e., the contribution of both modes vanishes, i.e., the integrals $S_k$ small, the beating term in (26) is proportional to. From this point of view, an increase of the modulation depth could be achieved by reducing the coupling coefficient $\kappa$ or the length of at least one section.

The modulation of the carrier densities during the beating pulsations is extremely weak and can be neglected as a small perturbation. It disappears in the limit of a large carrier life time $\tau$, whereas the frequency and amplitude of the power beating are independent of $\tau$. This behavior is the reason why extremely large pulsation frequencies can be achieved. It is completely different to the self Q-switching type of SP, which is caused by the changing carrier densities and slows down to zero in the limit of large $\tau$.

Some features of the up to 80 GHz self pulsations observed by Feiste et al. [2] in high-$\kappa L$ two-section DFB devices agree with our model. First, two main modes were observed, whose spectral difference coincides with the pulsation frequency. Second, each of these modes could be attributed to one of the two sections in accordance with the calculated spatial mode structure discussed above. Furthermore, the very fast SP of some tens of GHz were observed for more symmetric pump currents, whereas asymmetric pumping yielded the slower self Q-switching SP's. The following features, however, disagree completely. First, the theoretical beating frequency is independent on the currents, whereas the experimental one could be continuously tuned between 12 and 64 GHz by changing one of the currents between about 30 and 70 mA. Second, the laser used in the experiment has one and the same corrugation for both sections, i.e., $\delta x = 0$ holds, in which case no mode beating appears in the model. In our opinion, these discrepancies are - at least partly - due to the neglect of thermal effects in our example calculations. In practice, there is always a temperature difference due to the different heating of the differently pumped sections, which cannot follow the fast variations of the carrier concentrations. Thus, the corresponding thermally induced refractive index difference is equivalent to a static detuning $\delta x \approx (\pi/\lambda) \cdot (\delta \lambda/\lambda)$, where $\delta \lambda$ denotes the thermally induced wavelength shift. Estimating the latter one by $\approx 0.02$ nm/mA, as observed by Feiste et al. [2], one obtains roughly 1.5 cm$^{-1}$ static detuning per mA current difference, which is sufficiently large for the appearance of beating SP's. A detailed investigation of this effect is in work, preliminary results have been presented in [11].

Finally we note, that selfpulsations with similarly high repetition rates of several tens of GHz have also been reported by other authors for different types of devices. SP with frequencies of the order of 100 GHz have been theoretically

$$P_k(t) = P_{k1} + P_{k2} + 2\sqrt{P_{k1}P_{k2}} \text{Re}\{e^{i \Delta \omega t} S_k\},$$  \hspace{1cm} (26)
obtained by Shore and Rozzi [13] as well as Rahman and Winful [14] for stripe geometry Fabry–Perot lasers and laser arrays. They appeared if two lateral modes with the same gain coexisted. The lateral carrier density distribution remained essentially frozen during their evolution [14]. So far, these features are the lateral analogue to the mode beating between longitudinal modes discussed in this paper. Similar fast SP have also been reported for short external cavity lasers by Tager and Petermann [15], [16]. The mechanism behind these SP is again mode beating, because they appear in situations when two different compound cavity modes with identical gain coexist and their repetition rate corresponds well to the frequency spacing between these two modes. We conclude that all these fast SP are caused by similar mode beating effects.

VI. CONCLUSION

The system of coupled wave and carrier rate equations has been used for analyzing DFB lasers consisting of two independently pumped sections. As a new phenomenon not known from Fabry–Perot devices, the appearance of degeneration in the mode spectra has been found at certain values of the detuning between both sections. A mathematical investigation of the nonselfadjoint coupled wave operator has shown that the eigenfunctions (the longitudinal modes), which belong to a degenerate eigenvalue, span only a one dimensional subspace and, hence, do not describe all degrees of freedom. They must be accomplished by further functions, equations for which have been presented. On this base, the dynamics of a device is very helpful for a deeper insight into the modal dynamics. Here, we collect some main theorems, the proof of which will be presented elsewhere [28].

Concerning algebraically degenerated eigenvalues of the operators one may deduce the following result.

Theorem 2: Any vector $\Psi \in \mathcal{H}$ may be approximated by finite-linear combinations of vectors from the system $\Phi_m$, $\Omega_m \in \sigma(\mathcal{H}), k \in \{1, \ldots, l(m)\}$ and this is true not only in the original Hilbert space topology but also with respect to the norm

$$||\Psi|| = \left( \int_0^L \left| \frac{\partial \Psi_+}{\partial z} \right|^2 + \left| \frac{\partial \Psi_-}{\partial z} \right|^2 + |\Psi|^2 + |\Psi^-|^2dz \right)^{1/2},$$

which implies, in particular, uniform convergence on $[0, L]$.

We note that the above theorems hold generally for arbitrary functions $\beta(z)$, i.e., not only for constant sectional values of $\beta$ as in our example, but also when spatial hole burning is taken into account.

Let us still supply a remark on Fabry–Perot lasers. In FP lasers without internal reflections ($\kappa^2 = 0$), the complex mode frequencies can be analytically calculated to be

$$\nu_{m}^{FP} = -\frac{1}{L} \int_0^L \beta dz - \frac{i}{2L} \ln(r_0r_L),$$

where the mode index $m$ numbers the different sheets of the In function. Because these sheets are separated by $2\pi i$, the algebraic multiplicity of these eigenvalues is always one, i.e., no mode degeneracy may appear. Hence, the system of eigenfunctions (longitudinal modes) of FP lasers without internal reflections is complete.

REFERENCES


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